Solution: MATH 6635 Homework 5 due on 2/11/2015

Problem $\boxed{13}$: $A \in \mathbb{R}^{k \times n}$

$\because \|A\|_1 = \max \frac{\|Ax\|_1}{\|x\|_1} \qquad \|Ax\|_1 = \sum_{i=1}^{k} \left| \sum_{j=1}^{n} a_{ij} x_j \right| \le \sum_{i=1}^{k} |a_{ij}| \cdot \sum_{j=1}^{n} |x_j|$

① $\therefore \|A\|_1 = \max \frac{\|Ax\|_1}{\|x\|_1} \le \max \frac{\sum_{i=1}^{k} |a_{ij}| \cdot \sum_{j=1}^{n} |x_j|}{\sum_{j=1}^{n} |x_j|} = \max_{1 \le j \le n} \sum_{i=1}^{k} |a_{ij}|$

if: let $x = (0, \ldots 0, 1, 0, \ldots 0)^T$

then $\|Ax\|_1 = \sum_{i=1}^{k} |a_{ij}|$, $1 \le j \le n$

② $\therefore \|A\|_1 = \max \frac{\|Ax\|_1}{\|x\|_1} \ge \max_{1 \le j \le n} \sum_{i=1}^{k} |a_{ij}|$

Therefore: $\|A\|_1 = \max_{1 \le j \le n} \sum_{i=1}^{k} |a_{ij}|$ ☆

Problem $\boxed{14}$. Define: x_λ: the eigenvector of λ. $\Rightarrow$ $Ax_\lambda = \lambda x_\lambda$

x_β: — — — — of β. $\qquad Ax_\beta = \beta x_\beta$

$$\|A\| \|x_\lambda\| \ge \|Ax_\lambda\| = \|\lambda x_\lambda\| = |\lambda| \|x_\lambda\|$$

$$\therefore \|A\| \ge |\lambda|$$

$$Ax_\beta = \beta x_\beta$$

$$A^{-1} A x_\beta = A^{-1} \beta x_\beta$$

$$x_\beta = A^{-1} \beta x_\beta$$

$$\therefore A^{-1} x_\beta = \frac{1}{\beta} x_\beta$$

$$\therefore \|A^{-1}\| \|x_\beta\| \ge \|A^{-1} x_\beta\| = \left\| \frac{1}{\beta} x_\beta \right\| = \left| \frac{1}{\beta} \right| \|x_\beta\|$$

$$\therefore \|A^{-1}\| \ge \frac{1}{|\beta|}$$

$$\kappa(A) = \|A\| \|A^{-1}\| \ge |\lambda| \cdot \frac{1}{|\beta|} = \frac{|\lambda|}{|\beta|}$$ ☆

Problem 15:

$$\boldsymbol{x} = \begin{pmatrix} -8.5935 \\ 4.0732 \\ 0.2764 \\ 0.9187 \\ -1.1463 \end{pmatrix}$$