MATH 6635 <u>HW3</u>. due on 1/28/2015.

Solution

problem 9:

a): Based on Ito's lemma:

$X = W$. $dX = dW \Rightarrow \begin{cases} a = 0 \\ b = 1. \end{cases}$

$f(x,t) = x^2$: $\frac{\partial f}{\partial x} = 2x$ $\frac{\partial^2 f}{\partial x^2} = 2$ $\frac{\partial f}{\partial t} = 0$

$\therefore df = (a\frac{\partial f}{\partial x} + \frac{\partial f}{\partial t} + \frac{1}{2}b^2\frac{\partial^2 f}{\partial x^2})dt + b\frac{\partial f}{\partial x}.dW$

$= dt + 2x\,dW$

b): compute. $E[\cancel{d}W^2(T) - W^2(0)]$

$\because W(0) = 0$

$\therefore E[W^2(T) - W^2(0)] = E[W^2(T)] = Var(W(T)) + E[W(T)]^2.$

$\because W(T) - W(0) \sim N(0, T)$

$\therefore E[W^2(T) - W^2(0)] = Var[W(T)] + E[W(T)]^2 = T$

problem 10:

based on Ito's lemma:

$F(S,t) = S^n$ $\frac{\partial f}{\partial S} = nS^{n-1}$ $\frac{\partial^2 f}{\partial S^2} = n(n-1)S^{n-2}$. $\frac{\partial f}{\partial t} = 0$

$dS = \mu S dt + \sigma S dW$

$\therefore df = (a\frac{\partial f}{\partial S} + \frac{\partial f}{\partial t} + \frac{1}{2}b^2\frac{\partial^2 f}{\partial S^2})dt + b\frac{\partial f}{\partial S}.dW$

$= (\mu.S.n.S^{n-1} + \frac{1}{2}\sigma^2 S^2.n(n-1)S^{n-2})dt + \sigma S.nS^{n-1}.dW$

$= (\mu.n.S^n + \frac{1}{2}\sigma^2 n(n-1)S^n)dt + \sigma.n.S^n.dW$

$= n.S^n[\mu + \frac{1}{2}\sigma^2(n-1)]dt + \sigma nS^n dW$