MATH 6635 HW1: Solution

1. No arbitrage principle for American Call option

Consider the following strategy:

$$\left\{\begin{array}{ll} \text{1) borrow } V \text{ from Bank.} & +V \\ \text{2) buy the American call option at } V. & -V \end{array}\right] \text{initial costs} \quad 0$$

$$\left.\begin{array}{ll} \text{3) exercise the } \cancel{\text{option}} \text{ option to buy the underlying for } K. & \max\{S-K, 0\} \\ \text{4) return the borrowed money.} & -V \end{array}\right] \text{payoff. } S-K-V$$

based on No arbitrage principle. $\Rightarrow S-K-V \leq 0$.

$\therefore V \geq S-K.$

2 event A: students like chocolates

B: students like strawberries

$P(A)=0.6 \quad P(B)=0.7 \quad P(A\cap B)=0.4$

we want to know $P(A^c \cap B^c) = P(A\cup B)^c = 1-P(A\cup B)$

$= 1-[P(A)+P(B)-P(A\cup B)]$

$= 1-0.9 = 0.1$

3 $P(\text{other face is tail} \mid \text{toss is Head}) = \dfrac{P(\text{other face is tails} \cap \text{toss is Head})}{P(\text{toss is Head})}$

$= \dfrac{P(\text{other face is tail} \cap \text{toss is Head})}{P(\text{toss is H} \mid \text{two faces}) + P(\text{toss is H} \mid \text{two tails}) + P(\text{toss is H} \mid \text{fair coin})}$

$= \dfrac{\frac{1}{3}\times\frac{1}{2}}{1\times\frac{1}{3}+0+\frac{1}{2}\times\frac{1}{3}} = \dfrac{\frac{1}{6}}{\frac{3}{6}} = \dfrac{1}{3}$

4. No arbitrage principle: European put

Consider the following strategy:

$t\left\{\begin{array}{ll} \text{1) borrow } S+V \text{ at time } t \text{ from bank.} & +(S+V) \\ \text{2) buy the European put option at } V. & -V \\ \text{3) buy the underlying asset at } S. & -S \end{array}\right] \text{o Initial costs} = 0$

$T\left[\begin{array}{ll} \text{4) exercise the option to sell the underlying for } K. & +K \\ \text{5) return the borrow money at time } T & -(S+V)e^{r(T-t)} \end{array}\right.$

$K-(S+V)e^{r(T-t)} \leq 0$

$\Rightarrow \therefore V \geq Ke^{-r(T-t)} - S$