~~Short position: When we expect the value of the asset to go down. Thus writing calls or buying puts~~

~~Obs: Profit of holder at expiry = = Value of option at expiry − premium~~ x ~~$e^{r(T-t)}$~~

Compound ~~Simple~~ interest: A = ammount invested, r = annual interest, r constant 20% means $r = 0.2$. Let M be a positive integer. In $\frac{1}{M}$ of a year the interest earned is $\frac{r}{M}$. Assume after each $\frac{1}{M}$ of a year the interest earned is added to the principal and future interest is calculated on this quantity. Then, after $\frac{1}{M}$ of a year the principal is $A\left(1+\frac{r}{M}\right)$ after $\frac{2}{M}$ of a year $A\left(1+\frac{r}{M}\right)^2$ after a year $A\left(1+\frac{r}{M}\right)^M$ after t years $A\left(1+\frac{r}{M}\right)^{Mt}$

If $M=1$ it is called simple interest.
If $M>1$ it is called compounded {discretely} interest

$$\lim_{M\to\infty} A\left(1+\frac{r}{M}\right)^{Mt} = \lim_{M\to\infty} A\left(\left(1+\frac{r}{M}\right)^{\frac{M}{r}}\right)^{rt} =$$

Ae^{rt} called continuously compounded interest.

<u>Back to options.</u>

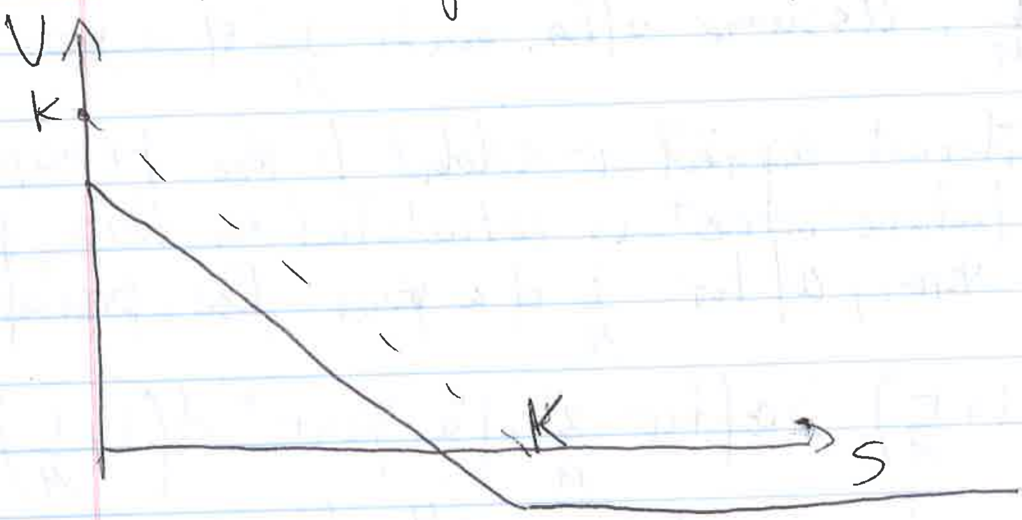
Profit of holder at expire = value of option at expire $-$ (Premium + transaction costs) $\times$ $e^{r(T-t)}$

this is $\to e^{r(T-t)}$ because the value of A at t is $Ae^{r(T-t)}$ at T (just put it in the bank)

t = time of transaction

T = expiry

<u>Ex</u>: Profit diagram of a put



Arbitrage: The existence of a portfolio, which requires no ~~investment initia~~ initial investment, risk free, i.e. guarantee no losses, but ~~very likely~~ gives gain at maturity.

No-arbitrage principle: This is an idealized assumption, frequently made, and that we will make, in the modeling of financial markets

Example: Let V_P^{Am} be the price of an American put at time t. Let K be the exercise price and $S = S(t)$ the spot price at time t. We will show that the no-arbitrage principle leads to

$$V_p^{Am} > \max\{K-S, 0\}$$

Consider the following strategy:

1) borrow $S + V_p^{Am}$
2) buy ~~the~~ put with V_p^{Am}
3) buy the underlying with S
4) exercise the option to sell the underlying for K
5) Return the borrowed money, $S + V_p^{Am}$

this is all done at once. ~~the~~ ~~profit is~~ If $K - S - V_p^{Am} > 0$, i.e. ~~this strategy gives a gua~~

$V_p^{Am} < K-S$, this is an arbitrage strategy, as it would lead to a guaranteed profit of $K-S-V_p^{Am}$.

the no arbitrage principle then implies

$V_p^{Am} \geq K-S$.

Obs: In practice, ~~I~~ if the ~~a~~ above situation ~~like~~ occurs, arbitrageurs, i.e. investors using arbitrage strategies, would follow the above strategy, thus buy ~~put~~ these american puts, which would bring the price V_p^{Am} up, ~~which will tend to eliminate this~~ ~~in~~ decreasing the profit $K-S-V_p^{Am}$, ~~tending~~ and eventually to eliminating the arbitrage. In ideal financial modeling, it is assume that this adjustment of the price of V_p^{Am}, happens ~~ins~~ in a very short time, instantaneously

Probability (review, /fast intro)

Sample space S = set of possible outcomes

Event E = a subset of S

$P(E)$ = probability event E occurs

Properties of P:

1) $0 \le P(E) \le 1 \quad \forall E \subset S$
2) $P(S) = 1$
3) $E_i \cap E_j = \emptyset \quad \forall i, j$, then

$$P(\bigcup E_i) = \sum P(E_i)$$

Prove that $P(A \cup B) + P(A \cap B) = P(A) + P(B)$

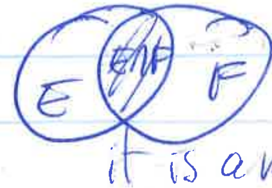
$\cup$ = union $\quad \sum$ = sum $\quad$ give example with dice

$A \cup B = (A - B) \oplus B \qquad A = A - B \oplus (A \cap B) \Rightarrow$ prove ...

$P(E/F)$ = prob E occurs given that F has occured

Obs: $P(E/F) = \dfrac{P(E \cap F)}{P(F)}$ $\quad$ give ex with dice

$\cap$ = intersection



If all events are equally probable it is a matter of counting

Def: E & F are independent if

$P(E|F) = P(E)$

Obs: E & F are independent $\iff$

$P(E \wedge F) = P(E) \times P(F)$