Final Friday, October 5, 2018 12:34 PM 1:33 PM Last name: First name: BU or BG:

Problem 1 (5 points). The family of differential equations $x' = x^3 - a x$ depends on a parameter a. Sketch the corresponding bifurcation diagrams.

Problem 2 (5 points). Find the fixed points and their type and using any of the techniques learned in class, draw the phase portrait of

x' = x(y + 2x - 2)y' = y(y - 1)

Problem 3 (5 points) Using any of the techniques learned in class, draw the phase portrait of

$$\begin{array}{l} x' = x + 2y \\ y' = -y \end{array}$$

Problem 4 (5 points): The system below is in polar coordinates. Draw the phase portrait of the system below, for values of a smaller, equal and bigger to all the values where a bifurcation occurs.

 $\begin{array}{l} r' = a \, r + r^3 \, - r^5 \\ \theta' = 1 \end{array}$