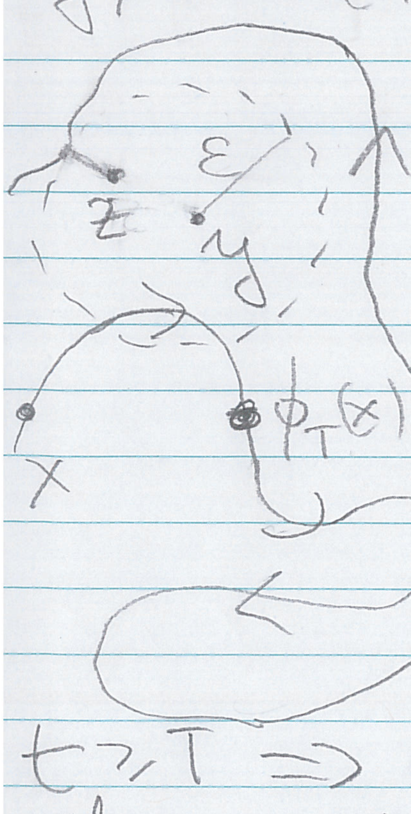
<u>Def</u>: S is a limit set if $S = \omega(x)$ for some x, or $S = \alpha(x)$ for some x.

<u>Prop</u>: $S = \omega(x)$, then S is a closed set

<u>proof</u>: $S = \omega(x)$. We are going to prove that S^c is open.

Let $y \notin S$, we need to show that there is $\varepsilon > 0$ such that $B_\varepsilon(y) \cap S = \phi$.

$y \notin S = \omega(x)$ implies that there exists $\varepsilon > 0$

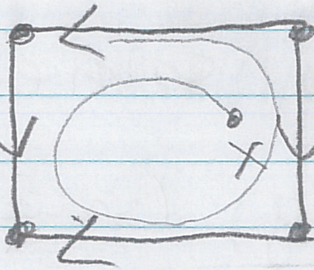


and T such that $\phi_t(x) \notin B_\varepsilon(y)$ for all $t \geq T$.

Let $z \in B_\varepsilon(y)$. We need to show that $z \notin S = \omega(x)$.

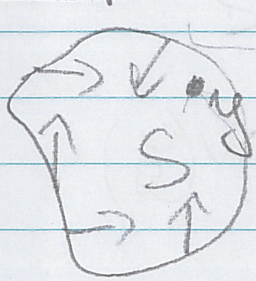
Since $\phi_t(x) \notin B_\varepsilon(y)$ for all $t \geq T \Rightarrow d(\phi_t(x), z) > \varepsilon - d(z, y) > 0$

then $z \notin \omega(x)$

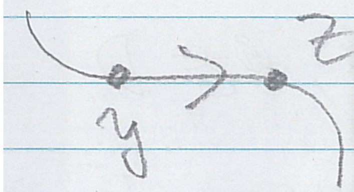
Ex: 

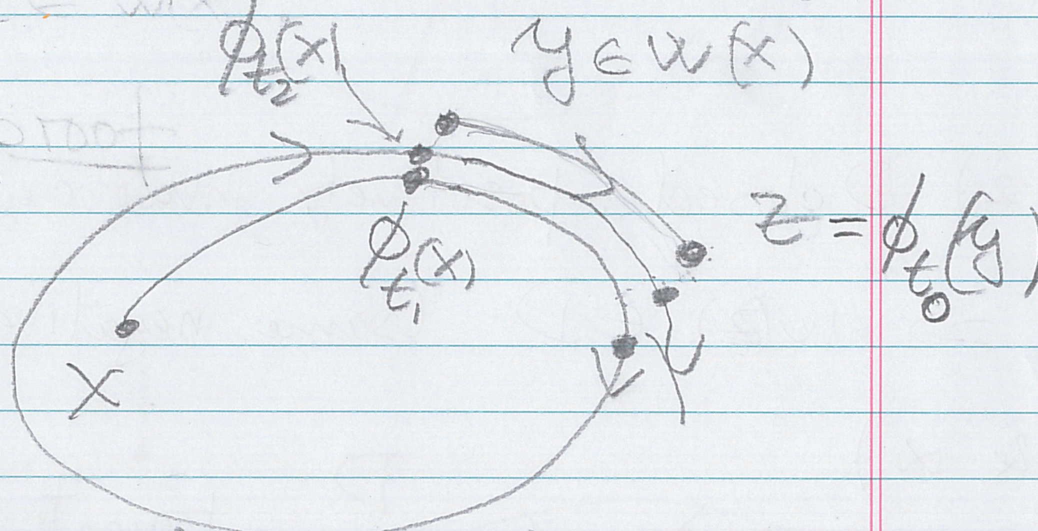
$w(x)$ is closed

Prop: $S = w(x) \Rightarrow S$ is invariant. proof Let $y \in S$. Let $t > 0$. Let $z = \phi_t(y)$. I want to show that $z \in S = w(x)$



$y \in w(x)$





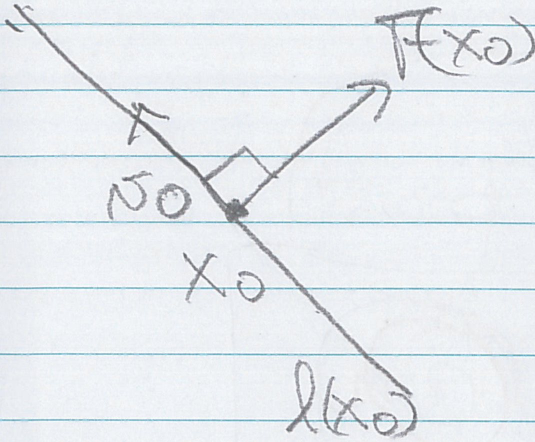
Since $y \in w(x)$, there exists $t_n \to \infty$ such that $\phi_{t_n}(x) \xrightarrow[n\to\infty]{} y$

$\phi_{t_n+t_0}(x) = \phi_{t_0}\left(\phi_{t_n}(x)\right) \xrightarrow[n\to\infty]{} \phi_{t_0}(y) = z$ because ϕ is a continuous and $\phi_{t_n}(x) \xrightarrow[n\to\infty]{} y$

thus $z \in w(x)$

Restrict ourselves to 2 dimensions

$x' = F(x)$. Let x_0 such $F(x_0) \neq 0$.



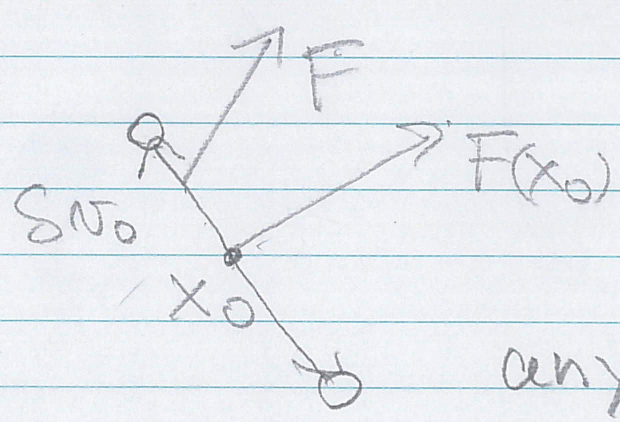
Let v_0 be a unit vector perpendicular to $F(x_0)$

$l(x_0) = \{ x_0 + r v_0 : \ r \in \mathbb{R} \}$

$h : \mathbb{R} \to l(x_0)$

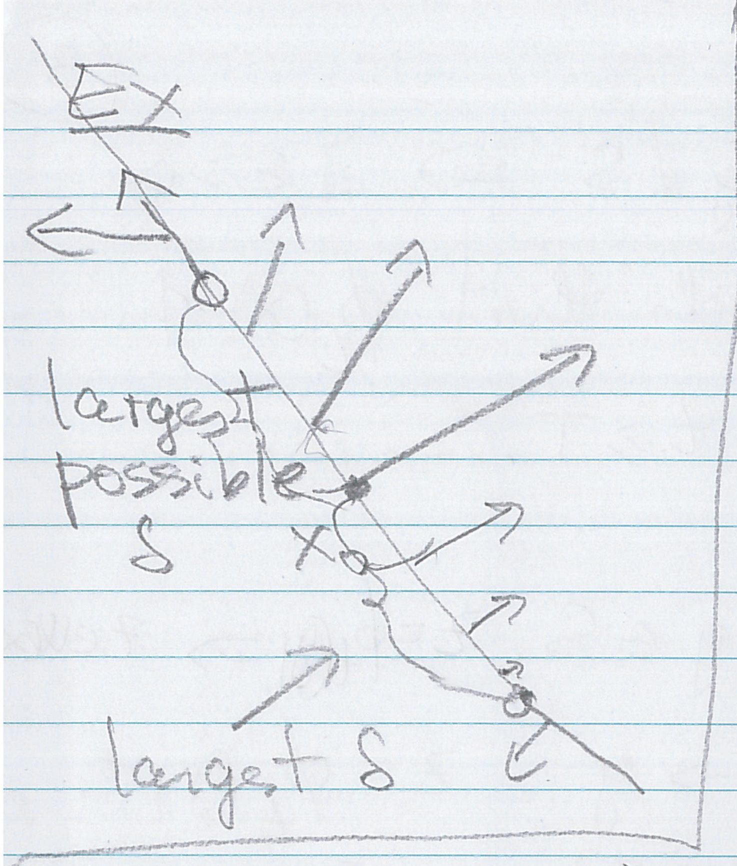
$h(r) = x_0 + r v_0$

Obs: Since F is continuous, there exists $\delta > 0$ such that

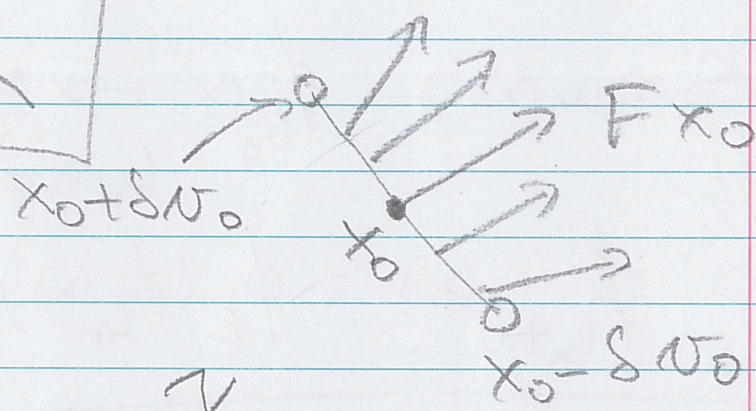


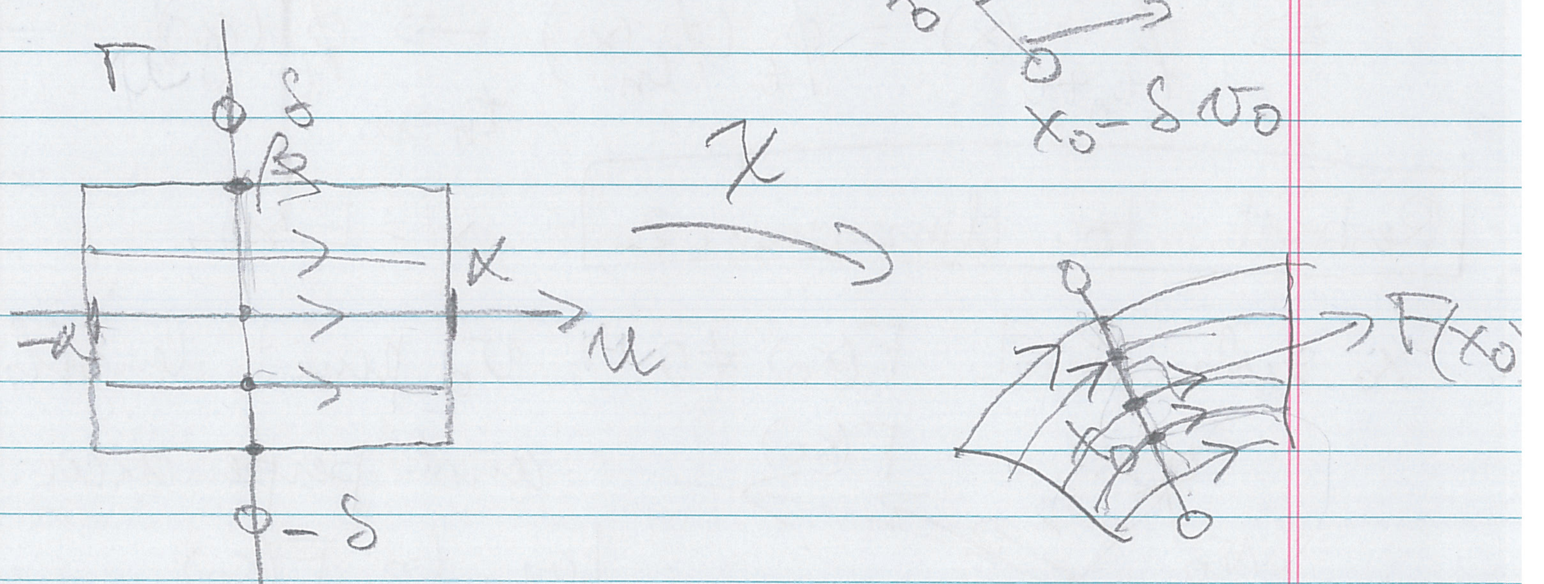
$F(h(r)) \neq \lambda v_0$ for any $\lambda \in \mathbb{R}$ and any $-\delta < r < \delta$.

Ex



Def: the segment $\{h(r) : -\delta < r < \delta\}$ is called a local section at x_0



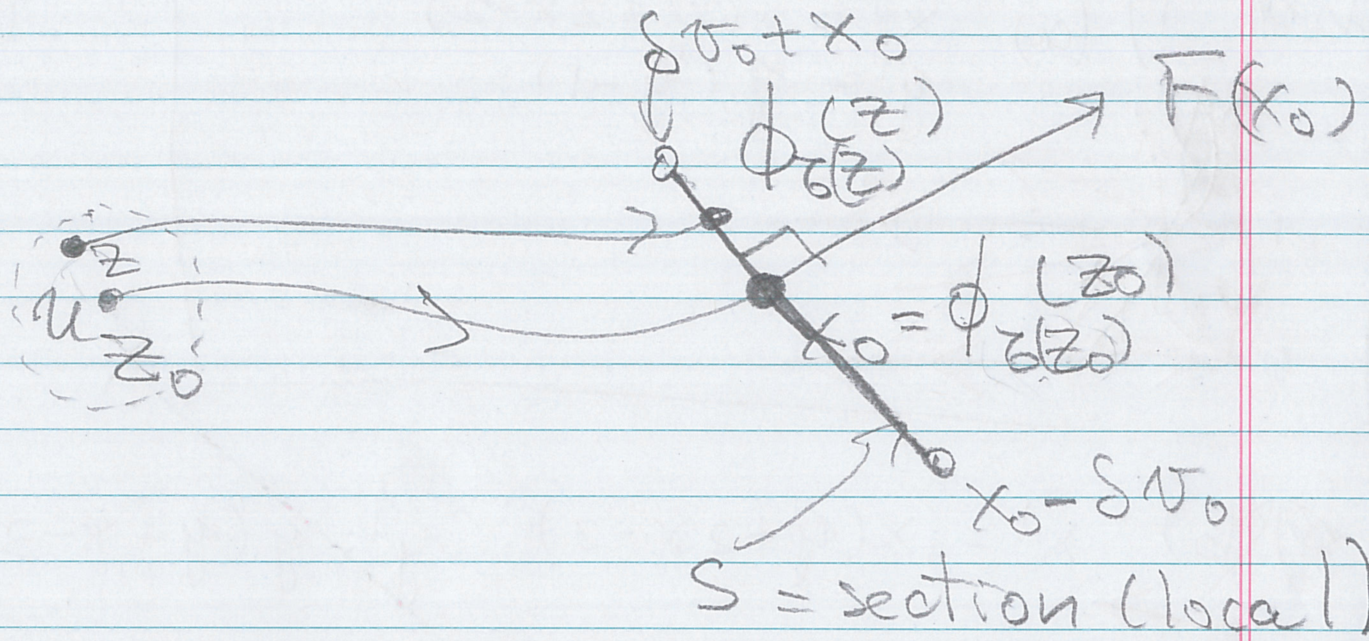


$\chi(u, r) = \phi_u(h(r)) \quad 0 < \beta < \delta$

$N = [-\alpha, \alpha] \times [-\beta, \beta]$

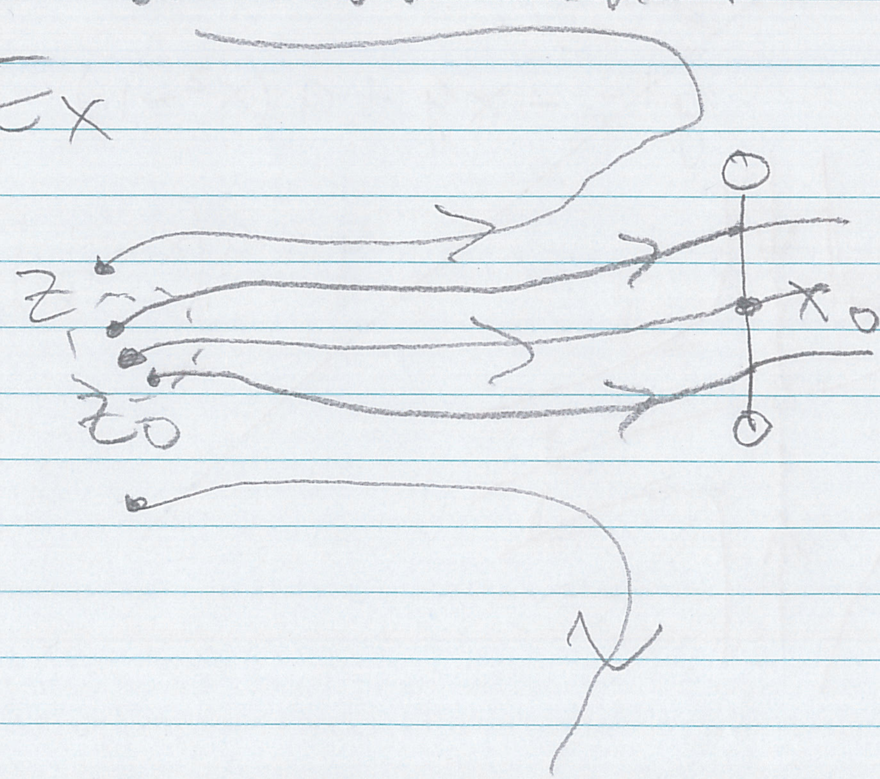
$\chi : N \longrightarrow \mathbb{R}^2$ is one to one

$\psi(N)$ is called a flow box.



Let z_0 such that x_0 is in the orbit of z_0. that is $x_0 = \phi_t(z_0)$ for some t. Call this $t = \tau(z_0)$.

Ex



Implicit function theorem:

$G : \mathcal{O} \to \mathbb{R}$ $\quad \mathcal{O} \subset \mathbb{R}^3$ $\quad \mathcal{O}$ open.

$x_0 \in \mathbb{R}^2$, $t_0 \in \mathbb{R}$ $\quad (x_0, t_0) \in \mathcal{O}$ $\quad G(x_0, t_0) = 0$

G is C^1. $\frac{\partial G}{\partial t}(x_0, t_0) \neq 0$. then, there exist $\mathcal{U}$ open in $\mathbb{R}^2$ such that $x_0 \in \mathcal{U}$ and $f : \mathcal{U} \to \mathbb{R}$ continuous such that $f(x_0) = t_0$ and $G(x, f(x)) = 0$ for all $x \in \mathcal{U}$.

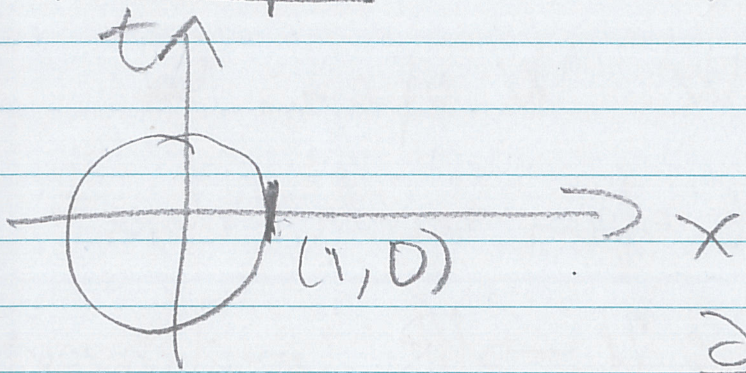
$G(x, t) = 0$ $\quad x \in \mathbb{R}^2$ $\quad t \in \mathbb{R}$

$G(x_0, t_0) = 0$ when we have $t = f(x)$ such that $t_0 = f(x_0)$ and $G(x, f(x)) = 0$? x is close to x_0, t will be closed to t_0, then

$$0 = G(x, t) \cong G(x_0, t_0) + \nabla_x G(x_0, t_0) \cdot (x - x_0) + \frac{\partial G}{\partial t}(x_0, t_0)(t - t_0) = 0$$

then $t = t_0 - \dfrac{\nabla_x G(x_0, t_0) \cdot (x - x_0)}{\dfrac{\partial G}{\partial t}(x_0, t_0)}$ 7

<u>Example</u> $G(x, t) = x^2 + t^2 = 1$



$$\frac{\partial G}{\partial t} = 2t$$

$$\frac{\partial G}{\partial t}(1, 0) = 0$$