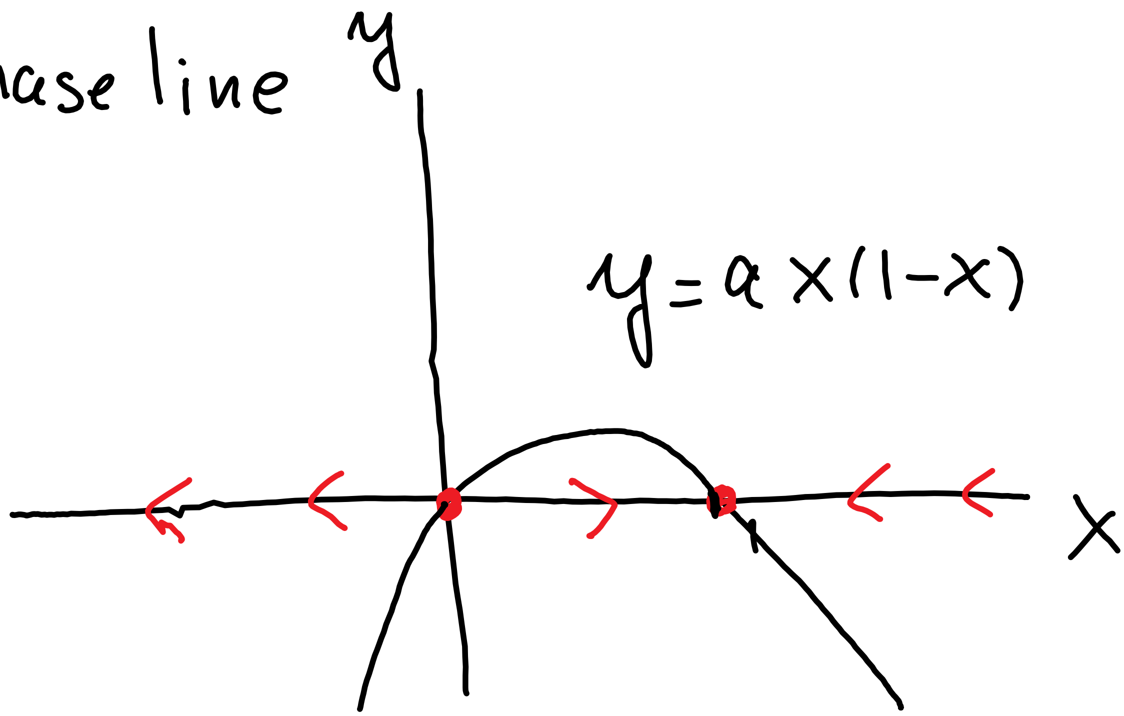
4541_2

Thursday, August 23, 2018 9:32 AM

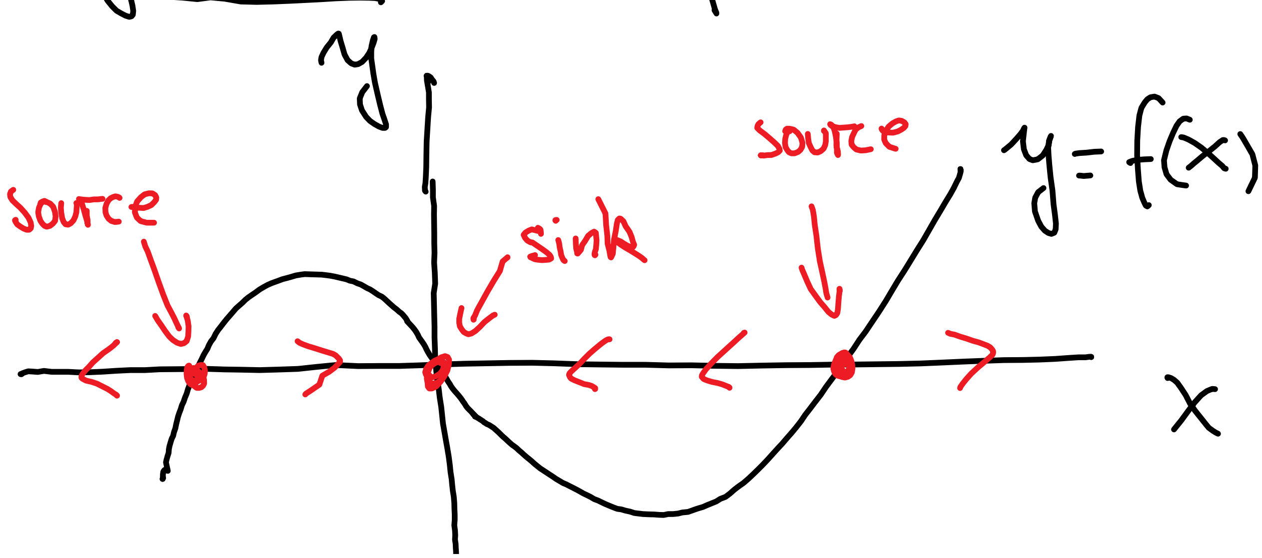
logistic model $x' = a\,x(1-x) = f(x)$

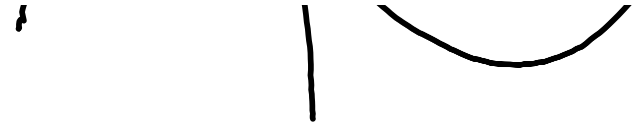
Phase line



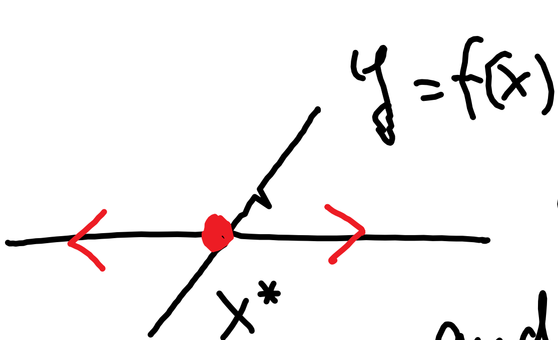
the phase line is the x-axis

<u>In general</u> $x' = f(x)$

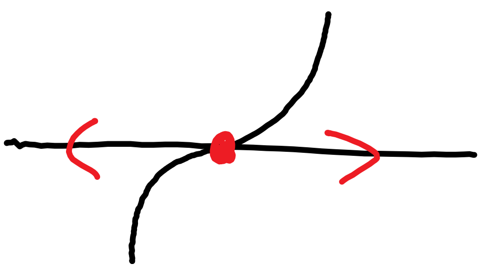




Blow up near a source



x^* is a source $\Longleftrightarrow f(x^*) = 0$ and f is increasing near x^* $\Longleftarrow f(x^*) = 0$ and $f'(x^*) > 0$



<u>Obs</u>: Let x^* be an equilibrium of $x' = f(x)$. Then:

1) If $f'(x^*) > 0 \Rightarrow x^*$ is a source
2) If $f'(x^*) < 0 \Rightarrow x^*$ is a sink
3) If $f'(x^*) = 0 \Rightarrow$ we do not know

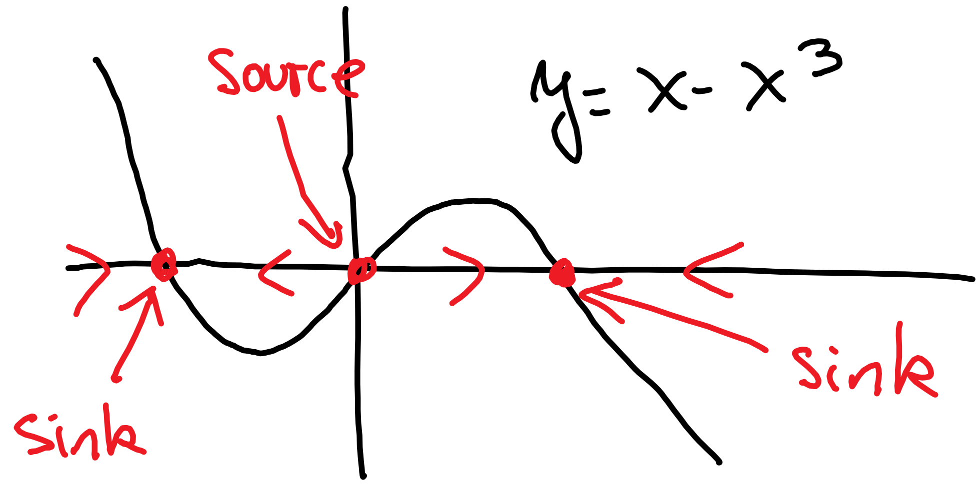
Example: $x' = x - x^3$. Find the equilibria of this equation and classify them as source, sink or neither.

$0 = x - x^3 = x(1-x)(1+x)$ $\quad f(x) = x - x^3$

Equilibria are $-1, 0, 1$

$f'(x) = 1 - 3x^2$

x^* = Equilibrium	sign of $f'(x^*)$	type
-1	$f'(-1) = -2 < 0$	sink
0	$f'(0) = 1 > 0$	source
1	$f'(1) = -2 < 0$	sink



<u>Theorem</u>: The initial value problem

$$x' = f(x,t)$$
$$x(0) = u$$

has unique solution for all $u \in \mathbb{R}$.

(This is true when f satisfies some conditions).

<u>Example</u>: $x' = x^{2/3}$

$$\int dx\, x^{-2/3} = \int dt$$

$$3x^{1/3} = t + c$$

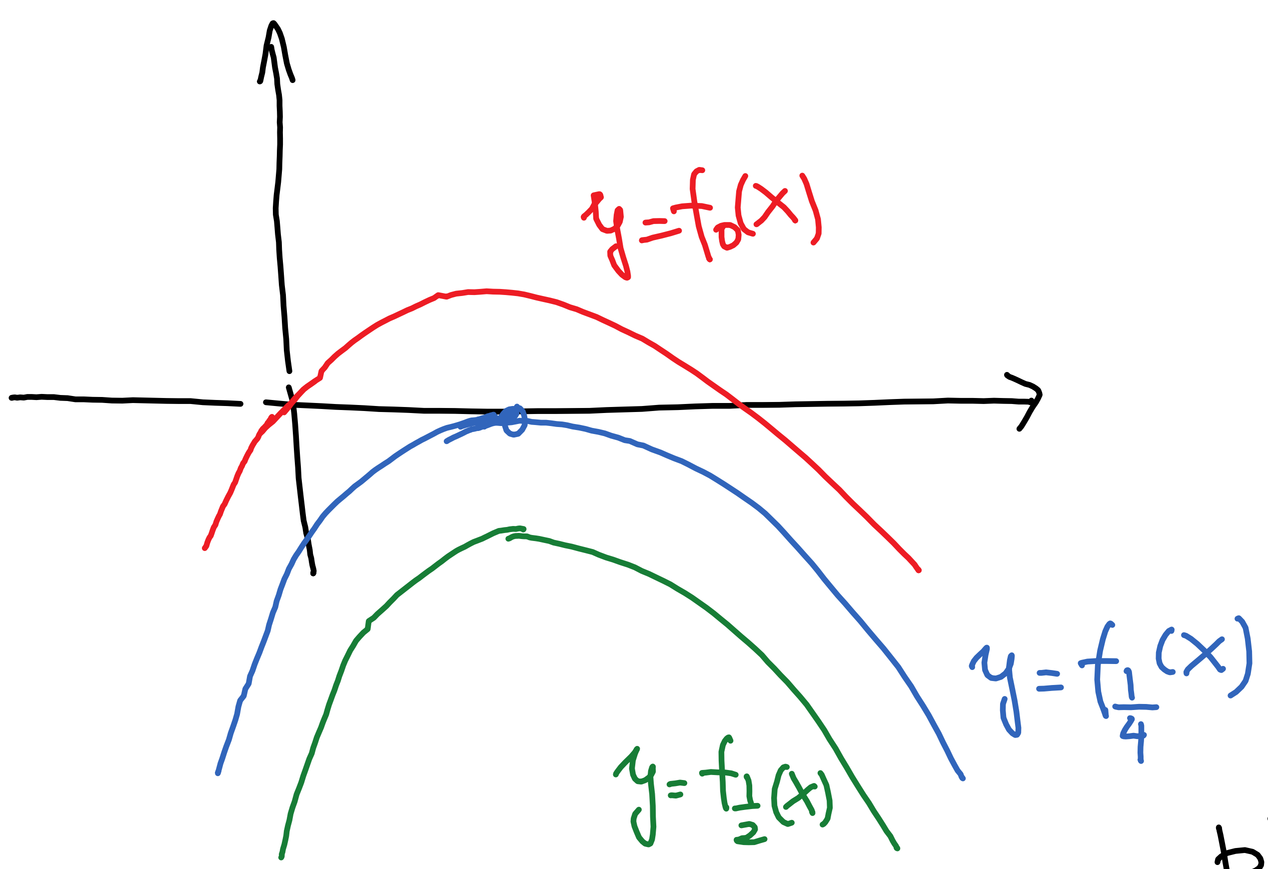
$$x = \frac{1}{27}(t+c)^3$$

(IVP) $x' = x^{2/3}$

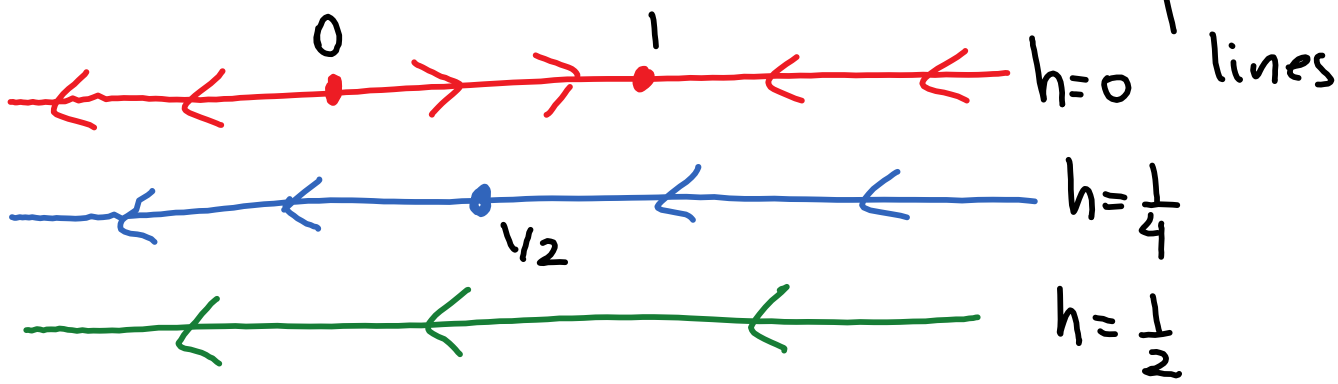
$x(0) = 0$

$x = \frac{1}{27}t^3$ and $x = 0$ are two solutions of the (IVP).

$\boxed{1.3}$ $x' = x(1-x) - h = f_h(x)$

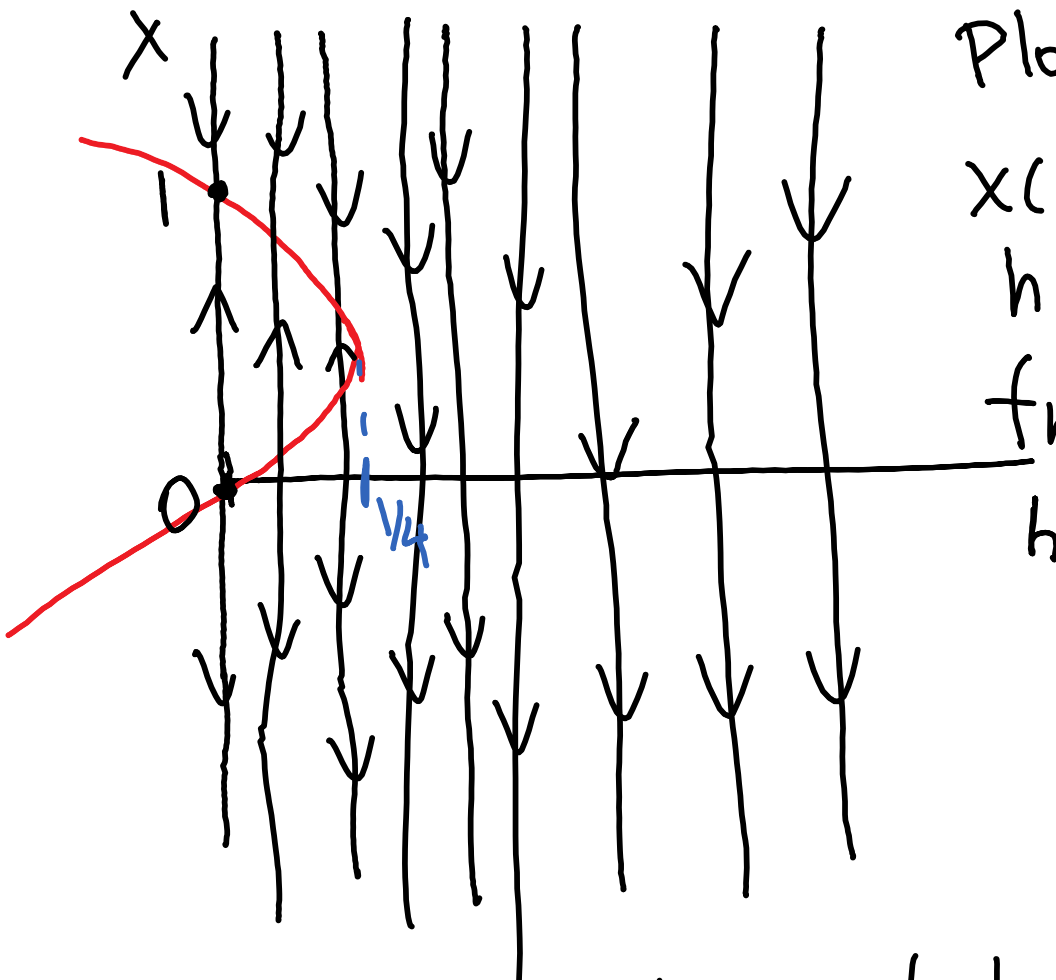


phase



Bifurcation diagram

Draw phase lines vertically. The horizontal axis is h.



Plot $f_h(x)=0$

$x(1-x)-h=0$

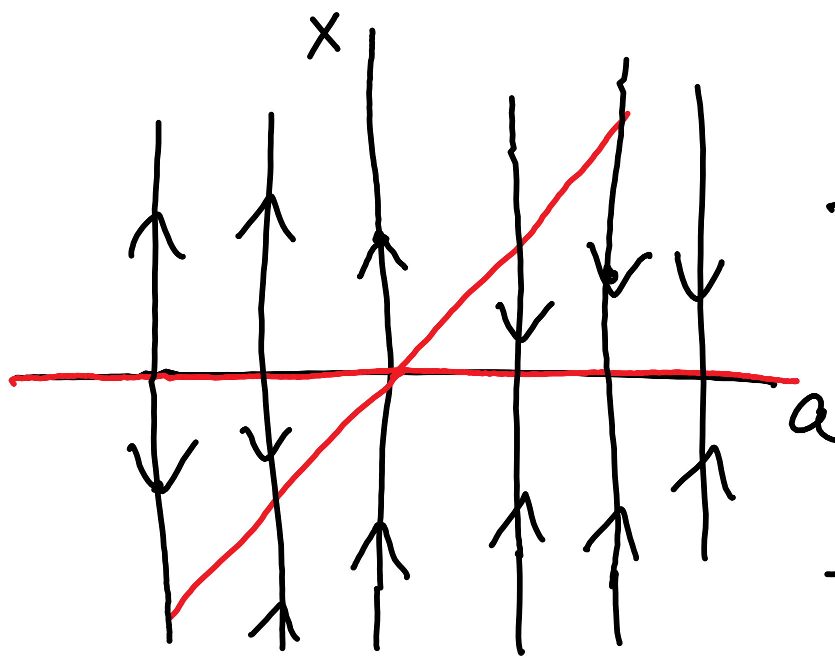
$h=x(1-x)$

$f_h(x)=x(1-x)-h$

There is a bifurcation at $h=\frac{1}{4}$. The qualitative behavior of the solutions changes as h goes from $h < \frac{1}{4}$ to $h > \frac{1}{4}$.

<u>Example</u> $x' = x(x-a)$

Draw the bifurcation diagram



$x(x-a) = 0$

$f(x) = x(x-a)$

$x = -1 \quad a = -2$

$f = (-1)(-1+2)$

$a = 2 \quad x = 1$

$f = -1 < 0$

We have a bifurcation at $a = 0$.

$\boxed{1.4}$ $x' = \underbrace{a x(1-x) - h(1+\sin(2\pi t))}_{f(t,x)}$

this is not an autonomous equation.

<u>Obs</u> $f(t+1,x) = f(t,x)$ for all t. f is 1-periodic on t.

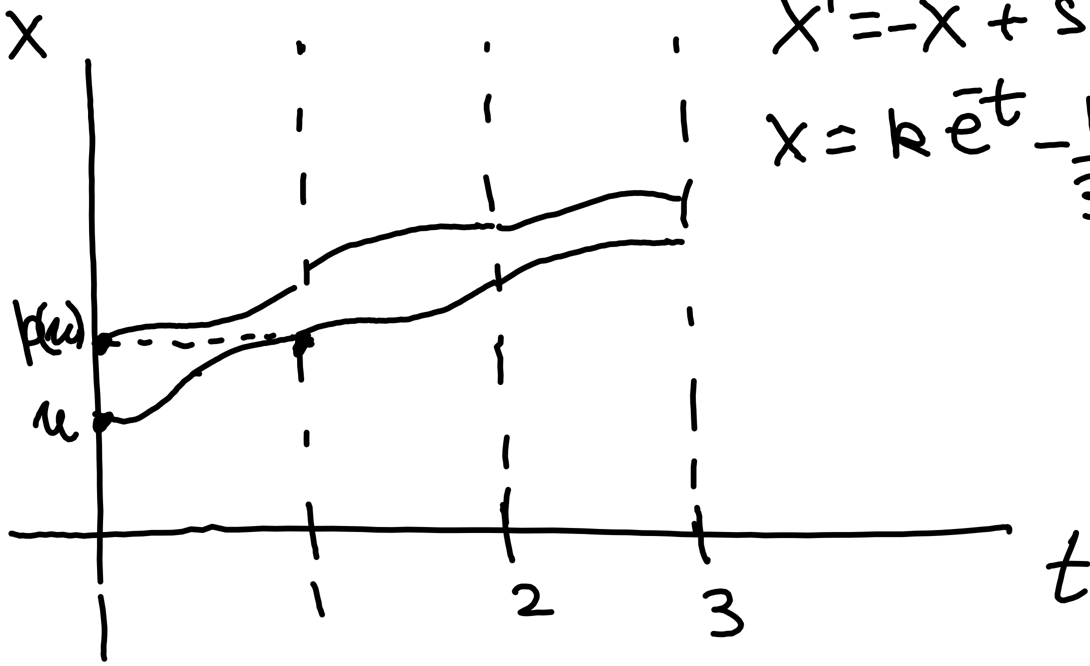
<u>Obs</u>: If x is a solution of $x' = f(x,t)$, so is $x_n(t) = x(t+n)$ for any $n \in \mathbb{Z}$

<u>check</u>: $\boxed{x_n'(t)} = x'(t+n) = f(t+n, x(t+n)) =$

$= \boxed{f(t, x_n(t))}$ ✓

$x' = -x + \sin(2t)$

$x = k e^{-t} - \frac{1}{3}\sin(2t)$



<u>Def</u>: $u \in \mathbb{R}$. Then $\phi(t,u)$ is the

Solution of $\dfrac{\partial \phi}{\partial t}(t,u) = f(t, \phi(t,u))$

$$\phi(0,u) = u$$

We define $p(u) = \phi(1,u)$