

Nullclines

$$x' = f(x, y)$$

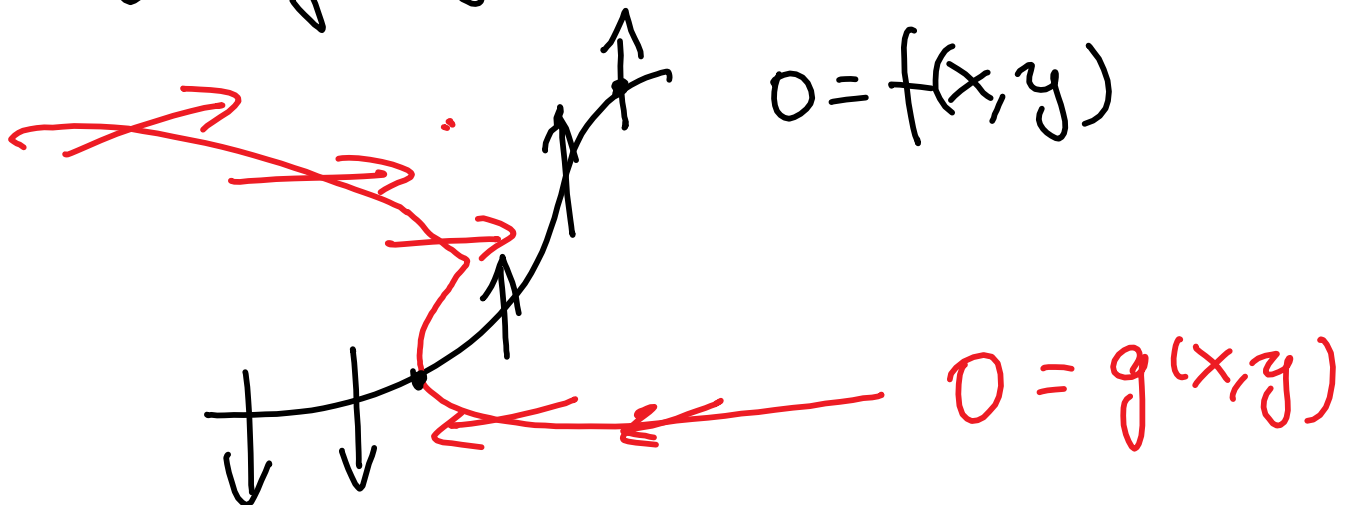
$$y' = g(x, y)$$

the x-nullcline is the curve

$$0 = f(x, y)$$

the y-nullcline is the curve

$$0 = g(x, y)$$



Example

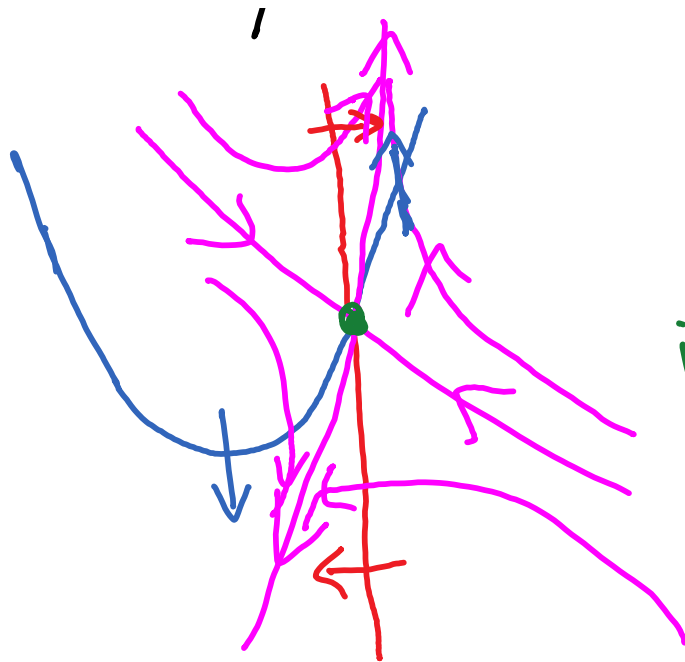
$$x' = y - x^2$$

$$y' = x - 2$$

$$y - x^2 = 0$$

$$x - 2 = 0$$

fixed point



• fixed point
(2, 4)
is a saddle

$$DF = \begin{bmatrix} -2x & 1 \\ 1 & 0 \end{bmatrix} \quad DF\left(\begin{bmatrix} 2 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} -4 & 1 \\ 1 & 0 \end{bmatrix}$$

$$(-4 - \lambda)(-\lambda) - 1 = 0 \quad \lambda^2 + 4\lambda - 1 = 0$$

$$\lambda = \frac{-4 \pm \sqrt{16 + 4}}{2} = -2 \pm \sqrt{5}$$

$$DF\left(\begin{bmatrix} 2 \\ 4 \end{bmatrix}\right) - (-2 + \sqrt{5})I = \begin{bmatrix} -2 - \sqrt{5} & 1 \\ 1 & 2 - \sqrt{5} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 - \sqrt{5} \end{bmatrix}(-2 - \sqrt{5}) = \begin{bmatrix} -2 - \sqrt{5} & (\sqrt{5})^2 - 4 \\ & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$V = \begin{bmatrix} 1 \\ 2 + \sqrt{5} \end{bmatrix}$$

Example

$$x' = x^2 - 1$$

$$y' = -xy + a(x^2 - 1)$$

x-nullcline

$$x^2 = 1$$

$$x = \pm 1$$

y-nullcline

$$0 = -xy + a(x^2 - 1)$$

$$y = \frac{a(x^2 - 1)}{x}$$

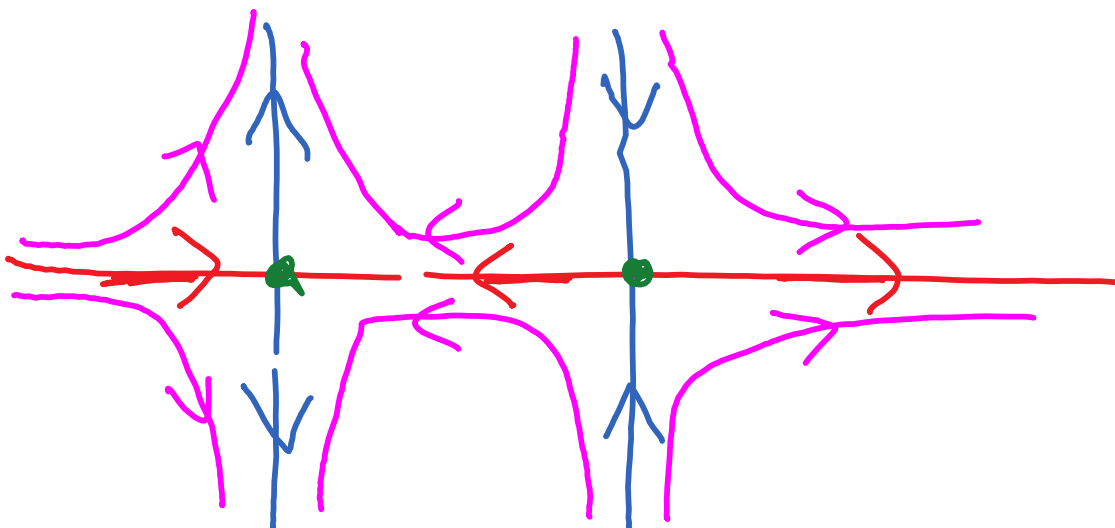
Case $a = 0$

$$x = \pm 1$$

x-n

$$y = 0$$

y-n



Case $a > 0$

$$x \rightarrow \infty \quad x = \pm 1$$

$$y \rightarrow \infty \quad y = a \frac{x^2 - 1}{x}$$

