Final - MATH 4320

Name

Problem 1 (5 points): Find all solutions to $z^4 = -8 - 8\sqrt{3}i$

Problem 2 (5 points): Sketch the following set $|2z + 3| > 4$.

Problem 3 (5 points): Let $\mathcal{C}$ be the semicircle $|z| = 2$ and $\mathrm{Im}(z) > 0$ in the counter-clockwise direction. Compute

$$\int_{\mathcal{C}} \frac{z+2}{z} dz$$

Problem 4 (5 points): Let $\mathcal{C}$ be the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant. Show that

$$\left| \int_{\mathcal{C}} \frac{dz}{z^2 - 1} \right| \leq \frac{\pi}{3}.$$

Problem 5 (5 points): Compute

$$\int_{|z|=1} \frac{dz}{z^2+2z+2}$$

Problem 6 (5 points): Let $\mathcal{C}$ be the positively oriented boundary of the square whose sides lie along the lines $x = -2$, $x = 2$, $y = 2$ and $y = -2$. Compute

$$\int_{\mathcal{C}} \frac{\cos z}{z^3 + 8z} dz.$$

Problem 7 (5 points): Suppose that $f(z)$ is entire and that the harmonic function $u(x, y) = \text{Re}[f(z)]$ has an upper bound u_0 for all points (x, y) in the xy plane. Show that $u(x, y)$ must be constant throught the plane.

Problem 8 (5 points): Find the Laurent series of

$$f(z) = \frac{z}{z^2 - (1+i)z + i}$$

in powers of $z - 1$ and give the region of convergence of each of the series.

Problem 9 (5 points): Compute

$$\int_{|z+2|=3} \frac{dz}{z^3(z+4)}$$

Problem 10 (5 points): Compute

$$\int_0^\infty \frac{dx}{x^4+1}$$