Homework 1 - MATH 4317

No collaboration. Due on Friday 9/29 during class. Show all your work.

Section AU or AG?:

Last name:

First name:

Problem 1 (5 points): Let $E = \{x \in \mathbf{R} : x \ge 1\}$. For all $x, y \in E$, we define d(x, y) = |1/x - 1/y|.

1) Prove that d is a distance in E. (1 points)

- 2) Is the sequence $x_n = n$ Cauchy with this distance? (Prove your answer). (2 points)
- 3) Does the sequence $x_n = n$ converge? (Prove your answer). (2 points)

Problem 2 (5 points): Show that the subset of \mathbf{R}^2 given by $\{(x, y) : y \ge |x|\}$ is closed.

Problem 3 (5 points): Prove that if $\lim_{n\to\infty} p_n = p$ in a given metric space then the set of points $\{p, p_1, p_2, p_3, \ldots\}$ is closed.

Problem 4 (5 points): Show that if a sequence of numbers a_1, a_2, a_3, \ldots converges to a, then $\lim_{i \to \infty} \sum_{i=1}^{n} a_i$

$$\lim_{n \to \infty} \frac{\sum_{i=1}^{n} a_i}{n} = a$$