Final - MATH 4317

Last name:

First name:

Problems you want graded: (Select exactly 4)

Each problem is worth 5 points

Problem 1: Prove that if $f : [a, b] \to \mathbb{R}$ is increasing then $\int_a^b f(x)dx$ exists.

Problem 2: Prove that a differentiable real-valued function on $\mathbb{R}$ with bounded derivative is uniformly continuous.

Problem 3: Let $f, f_1, f_2, f_3, \cdots$ be continuous real-valued functions on a compact metric space E, with $f(x) = \lim_{n\to\infty} f_n(x)$ for all $x \in E$. Prove that, if $f_1(x) \leq f_2(x) \leq f_3(x) \cdots$ for all $x \in E$, then the sequence $f_1, f_2, f_3, \cdots$ converges uniformly.

Problem 4: Let E be metric space with more than one point. Let $x_0 \in E$. Assume $\{x_0\}$ is open. What can you say about the connectivity of E?

Problem 5: Let $f : \mathbb{R} \to \mathbb{R}$ be a function which is strictly increasing and onto. Prove that f and f^{-1} are continuous.

Problem 6: Show that if $f : E \to E'$ and $g : E' \to E''$ are both uniformly continuous, so is the composition $gof : E \to E''$