## Extra credit problems - MATH 4317

No collaboration. Show all your work. submit the problems on the day they are due. The problems are due on different days.

Section AU or AG?:

Last name:

First name:

**Problem 1 (due Monday 10/23):** Let E be a metric space. Let A and B be two closed sets in E. Assume A and B do not intersect. Prove that there exist two open sets U and V such that: 1) A is included in U, 2) B is included in V and 3) U and V do not intersect.

**Problem 2 (due Wednesday 10/25):** Let *E* be a metric space. Let  $a \in E$ . For each positive integer *k*, let  $a_n^{(k)}$  be a sequence in *E* that converges to *a* as  $n \to \infty$ . Let  $x_n = a_n^{(n)}$ . Question: Does  $x_n$  converge? If yes, prove it. If no, give a counterexample.

**Problem 3 (due friday 10/27):** In  $\mathbb{R}^n$ , bounded and closed implies compact. Give an example of a complete metric space E and a set S in E that is bounded and closed, but not compact. Prove your claims.

**Problem 4 (due Monday 10/30):** A metric space E is said to be locally connected if for all  $x \in E$ , there exists  $\varepsilon > 0$  such that  $B_{\varepsilon}(x)$  is connected. Show that if E is locally connected, then E is the disjoint union of open connected sets.