Homework 2 - MATH 4317

Five problems from this list will be in the final.

Problem 1 (5 points): Assume that a metric space E has the property that the intersection of any collection of open sets is open. Which subsets of E are open?

Problem 2 (5 points): Let U, V be open intervals in $\mathbb{R}$. Let $f : U \to V$ be a function which is strictly increasing and onto. Prove that f and f^{-1} are continuous.

Problem 3 (5 points): Let E be metric space with more than one point. Let $x_0 \in E$. Assume $\{x_0\}$ is open. Is E connected? Prove it.

Problem 4 (5 points): Assume that a metric space E is complete and has the following property:

If x_n is a sequence and there exists $\varepsilon > 0$ such that $d(x_i, x_j) > \varepsilon$ for all $i \neq j$, then the sequence x_n is not bounded.

Prove that if $S \subset E$ is bounded and closed, the S is compact. (Hint. You can use this fact: If every sequence has a convergent subsequence, then E is compact)