MATH 4305 - Homework 2 - Due 11/10

AU or AG:

Last Name:

First Name:

Justify your answers

Problem 1 (5 points): Let V be the set of polynomials p of degree less or equal to 3 (including the polynomial 0) such that $p(1) = 0$. Is V a linear space? If the anwser is yes, what is the dimension of V? and find a basis of V.

Problem 2 (5 points): Let V be the set of polinomials of degree less or equal to 2 (including the polynomial 0). For each $p \in V$, we define

$$T(p) = \begin{bmatrix} p(-1) \\ p'(0) \\ p(0) \\ p'(1) \end{bmatrix},$$

where p' is the derivative of p.

a) (1 point) Is T a linear transformation?

b) (2 point) If yes above, find a basis of the kernel of T?

c) (2 point) If yes in a), find a basis of the image of T.

Problem 3 (5 points): Perform the Gram-Schmidt process on the sequence of vectors

$$\begin{bmatrix} 1 \\ 7 \\ 1 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 7 \\ 2 \\ 7 \end{bmatrix}, \begin{bmatrix} 1 \\ 8 \\ 1 \\ 6 \end{bmatrix}.$$

Problem 4 (5 points): Find the least square solution of ths system $Ax = b$, where

$$A = \begin{bmatrix} 3 & 2 \\ 5 & 3 \\ 4 & 5 \end{bmatrix}, \text{ and } b = \begin{bmatrix} 5 \\ 9 \\ 2 \end{bmatrix}.$$